IDEAL OF PRIME GAMMA RINGS WITH LEFT DERIVATIONS

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Abstract: Let M be a prime Γ-ring, U is a ideal of M and d be a nonzero left derivation of M. If $[d(y), d(x)]_\alpha = [y, x]_\alpha$ such that for all $x, y \in U$ and $\alpha, \beta \in \Gamma$, then $[x, d(x)]_\alpha = 0$ and hence M is a commutative.

Keywords: Derivation, Left derivation, Prime Γ-ring, Prime ring.

1. Introduction: The concept of the Γ-ring was first introduced by Nobusawa [8] and also shown that Γ-rings, more general than rings. Bresar and Vakman [2] studied on some additive mapping in rings with involution. Barnes [1]studied on the gamma rings of Nobusawa. Ceven [3] studied on Jordan left derivations on completely prime Γ-rings. Mayne [7] have developed some remarkable result on prime rings with commuting and centralizing. Luh [6] studied on the theory of simple gamma rings. Jaya Subba Reddy.C et al. [4] studied centralizing and commutating left generalized derivation on prime ring is commutative. Jaya Subba Reddy. C et al. [5] studied the right reverse derivation on prime ring is commutative. Salah Mehdi Salih et al. [9] studied on ideal of prime gamma rings with right reverse derivations. In this paper, we extended some results on ideal of prime gamma rings with left derivations.

2. Preliminaries

Let M and Γ be additive abelian groups. If there exists a mapping $(x, \alpha, y) \to x\alpha y$ of $M \times \Gamma \times M \to M$, which satisfies the conditions

(i) $x\alpha y \in M$

(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)z = x\alpha z + x\beta z$, $x\alpha(y + z) = x\alpha y + x\alpha z$

(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring.

A Γ-ring M is said to be prime if $a\Gamma M\Gamma b = (0)$ with $a, b \in M$, implies $a = 0$ or $b = 0$. If M is a Γ-ring, then $[x, y]_\alpha = x\alpha y - y\alpha x$ is known as the commutator of x and y with respect to α, where $x, y \in M$ and $\alpha \in \Gamma$. We make the basic commutator identities:

$[x\alpha y, z]_\beta = [x, z]_\beta \alpha y + x\alpha[y, z]_\beta$ and $[x, y\alpha z]_\beta = [x, y]_\beta \alpha z + y\alpha[x, z]_\beta$, for all $x, y \in M$ and $\alpha \in \Gamma$. We consider the following assumption:

(A).................$x\alpha y\beta z = x\beta y\alpha z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. An additive mapping $d: M \to M$ is called a derivation if $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$ holds for all $x, y \in M$ and $\alpha \in \Gamma$. An additive mapping $d: M \to M$ is called a left derivation if $d(x\alpha y) = x\alpha d(y) + y\alpha d(x)$ holds for all $x, y \in M$ and $\alpha \in \Gamma$.

3. Main Results:

Theorem 3.1: Let M be a prime Γ-ring, U is a nonzero ideal of M and d is a left derivation of M, if U is a non-commutative such that (A) for all $x, y, z \in U$ and $\alpha, \beta \in \Gamma$, then $d = 0$.

Proof: Since d is a left derivation and since (A) then

Let $d(y\beta x\alpha x) = y\beta x\alpha d(x) + x\alpha y\beta d(x) + x\alpha x\beta d(y)$. (1)

On other hand

$$d\big(y\beta(x\alpha x)\big) = y\beta d(x\alpha x) + x\alpha x\beta d(y)$$

$= y\beta x\alpha d(x) + y\beta x\alpha d(x) + x\alpha x\beta d(y)$. (2)

Comparing equations (1) and (2), using (A), we get
$x\alpha y\beta d(x) = y\beta x\alpha d(x)$
$x\alpha y\beta d(x) = y\alpha x\beta d(x)$
$x\alpha y\beta d(x) - y\alpha x\beta d(x) = 0$
$(x\alpha y - y\alpha x)\beta d(x) = 0$
$[x, y]_\alpha \beta d(x) = 0$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$. (3)
We replace y by $z\beta y$ in equation (3), and using equation (3), we get
$[x, z\beta y]_\alpha \beta d(x) = 0$
$z\beta[x, y]_\alpha \beta d(x) + [x, z]_\alpha \beta y \beta d(x) = 0$
$[x, z]_\alpha \beta y \beta d(x) = 0$, for all $x, y, z \in U$ and $\alpha, \beta \in \Gamma$. (4)
Replacing y by $m\alpha y$, $m \in M$ in equation (4), we get
$[x, z]_\alpha \beta\ m\alpha y \beta d(x) = 0$, for all $x, y, z \in U$, $\alpha, \beta \in \Gamma$ and $m \in M$.
If we interchange m and y then, we get
$[x, z]_\alpha \beta\ y\alpha m \beta d(x) = 0$, for all $x, y, z \in U$, $\alpha, \beta \in \Gamma$ and $m \in M$.
By primness property, either $[x, z]_\alpha = 0$ or $d(x) = 0$.
Since U is a non-commutative, then $d = 0$.

Theorem 3.2: Let M be a prime Γ-ring, U is a ideal of M and d be a nonzero left derivation of M. If $[d(y), d(x)]_\alpha = [y, x]_\alpha$ such that (A) for all $x, y \in U$ and $\alpha, \beta \in \Gamma$, then $[x, d(x)]_\alpha = 0$ and hence M is a commutative.
Proof: Given$[d(y), d(x)]_\alpha = [y, x]_\alpha$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$.
Replacing y by $x\beta y$ in above equation, we get
$[d(x\beta y), d(x)]_\alpha = [x\beta y, x]_\alpha$
$x\beta[y, x]_\alpha + [x, x]_\alpha \beta y = [d(x\beta y), d(x)]_\alpha$
$x\beta[y, x]_\alpha = [x\beta d(y) + y\beta d(x), d(x)]_\alpha$
$x\beta[y, x]_\alpha = \big(x\beta d(y) + y\beta d(x)\big)\alpha d(x) - d(x)\alpha(x\beta d(y) + y\beta d(x))$
$x\beta[y, x]_\alpha = x\beta d(y)\alpha d(x) + y\beta d(x)\alpha d(x) - d(x)\alpha x\beta d(y) - d(x)\alpha y\beta d(x)$
Adding and subtracting $x\alpha d(x)\beta d(y)$ then, we get
$x\beta[y, x]_\alpha = x\beta d(y)\alpha d(x) + y\beta d(x)\alpha d(x) - d(x)\alpha x\beta d(y) - d(x)\alpha y\beta d(x) - x\alpha d(x)\beta d(y) + x\alpha d(x)\beta d(y) x\beta[y, x]_\alpha = x\beta d(y)\alpha d(x) + y\alpha d(x)\beta d(x) - d(x)\alpha x\beta d(y) - d(x)\alpha y\beta d(x) - x\beta d(x)\alpha d(y) + x\alpha d(x)\beta d(y) x\beta[y, x]_\alpha = x\beta d(y)\alpha d(x) - x\beta d(x)\alpha d(y) + y\alpha d(x)\beta d(x) - d(x)\alpha y\beta d(x) + x\alpha d(x)\beta d(y) - d(x)\alpha x\beta d(y)\ \ x\beta[y, x]_\alpha = x\beta[d(y), d(x)]_\alpha + [y, d(x)]_\alpha \beta d(x) + [x, d(x)]_\alpha \beta d(y)$
Using hypothesis then, we get
$[y, d(x)]_\alpha \beta d(x) + [x, d(x)]_\alpha \beta d(y) = 0.$ (5)
Replacing y by $y\alpha c$, where $c \in Z(M)$ and using equation (5), we get
$[y\alpha c, d(x)]_\alpha \beta d(x) + [x, d(x)]_\alpha \beta d(y\alpha c) = 0$
$y\alpha[c, d(x)]_\alpha \beta d(x) + [y, d(x)]_\alpha \alpha c \beta d(x) + [x, d(x)]_\alpha \beta\big(y\alpha d(c) + c\alpha d(y)\big) = 0$
$y\alpha[c, d(x)]_\alpha \beta d(x) + [y, d(x)]_\alpha \alpha c \beta d(x) + [x, d(x)]_\alpha \beta y \alpha d(c) + [x, d(x)]_\alpha \beta c \alpha d(y) = 0$
$y\alpha[c, d(x)]_\alpha \beta d(x) - [x, d(x)]_\alpha \alpha c \beta d(y) + [x, d(x)]_\alpha \beta y \alpha d(c) + [x, d(x)]_\alpha \beta c \alpha d(y) = 0$
$y\alpha[c, d(x)]_\alpha \beta d(x) - [x, d(x)]_\alpha \beta c \alpha d(y) + [x, d(x)]_\alpha \beta y \alpha d(c) + [x, d(x)]_\alpha \beta c \alpha d(y) = 0$
$[x, d(x)]_\alpha \beta y \alpha d(c) = 0$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$.
Since $0 \neq d(c) \in Z(M)$ and U is a ideal of M, then we have $[x, d(x)]_\alpha = 0$, for all $x \in U$.
By using the similar procedure as in Theorem (3.1), then we get either $[x, z]_\alpha = 0$ or $d(x) = 0$. Since d is a nonzero, then$[x, z]_\alpha = 0$. Hence M is a commutative.

Theorem 3.3: Let M be a prime Γ-ring, U is a ideal of M and d be a nonzero left derivation of M. If $[d(y), d(x)]_\alpha = 0$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$, then M is a commutative.
Proof: Given that$[d(y), d(x)]_\alpha = 0$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$.
Replacing y by $x\beta y$ in above equation then, we get
$[d(x\beta y), d(x)]_\alpha = 0$, for all $x, y \in U$ and $\alpha, \beta \in \Gamma$.

$[x\beta d(y) + y\beta d(x), d(x)]_\alpha = 0$
$[x\beta d(y), d(x)]_\alpha + [y\beta d(x), d(x)]_\alpha = 0$
$x\beta[d(y), d(x)]_\alpha + [x, d(x)]_\alpha \beta d(y) + y\beta[d(x), d(x)]_\alpha + [y, d(x)]_\alpha \beta d(x) = 0$
$$[y, d(x)]_\alpha \beta d(x) + [x, d(x)]_\alpha \beta d(y) = 0. \qquad (6)$$
The proof is now completed by equation (5) of Theorem (3.2). Hence M is a commutative.

Acknowledgment: Financial support is nil.

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